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RELAXATION METHOD OF SOLVING THE CIRCUIT OF AN INDUCTION MOTOR WITH A PHASE ADVANCER

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(*Received May* 18, 1966)

ABSTRACT. In this paper it has been discussed how the relaxation method can be suitably applied to solve the circuit problem of an induction motor with a phase advancer For this purpose the relaxational technique of solution of A C. networks with complex circuit constants is used It also discusses a method slightly different from the existing ones for obtaining operation groups to liquidate the residuals of linear simultaneous equations in a definite number of steps. The results obtained thereby are compared with those found out by conventional method of solution of A C. networks which is illustrated with an example

INTRODUCTION

The basic idea of having a phase advancer consists in so adding to the rotor induced e m.f. that there is a phase advancement in the rotor current with a consequent reaction on the stator to advance the phase of the stator current also (Say, 1962). The equivalent circuit of an induction motor with a phase advancer is shown in Fig. 1 (Mem. Staff. Dept. Elect. Engg., M I T., 1953), which can be transformed into a suitable form shown in Fig. 2a Relaxation method can be conveniently applied to solve this network yielding the values of many desired quantities simultaneously. Southwell and Black (1938) have used the relaxation method to solve the problem of A C networks and have shown that the presence of complex circuit constants does not add much difficulty in getting its relaxational solution The advantages of the method will be indicated by solving the problem represented in the Fig. 2a.

To liquidate the residuals in the solution of linear simultaneous equations a method with some difference from those suggested by Bandyopadhyay and Narshimhan (1956), and Basu (1958) has been developed. In this method the difference lies in the fact that operation groups (Allen, 1954), are found out which will keep only one residual at a time i.e. 1st, 2nd, 3rd etc , successively, unchanged From this set of group other set of operation groups can be obtained to keep a number of residuals, viz., 1st and 2nd; 1st, 2nd and 3rd and so on, unchanged simultaneously. With the choice of suitable multiples for these groups all the residuals can be liquidated in succession, the maximum number of steps being equal to the number of the residuals.

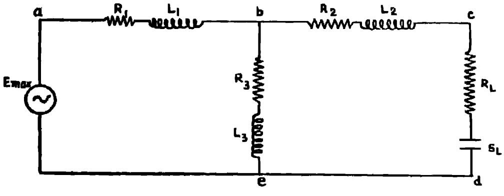

Fig. 1

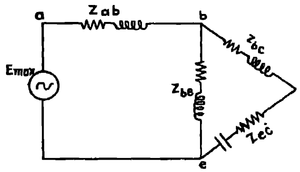

Fig. 2 a

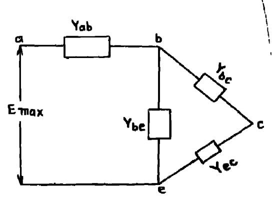

Fig. 2 b

THE METHOD

Let Z_{ab}, Z_{bc} ... etc., be the impedances and Y_{ab}, Y_{bc} ... etc , be the corresponding admittances of the respective branches of the network shown in Fig. 2b. Considering the admittance of a branch such as bc it may be written as :

$$Y_{bc} = g_{bc}+jb_{bc} \tag{1}$$

and if the potentials at the nodal points b and c be :

$$\left.\begin{aligned} V_b &= v_{x(b)}+jv_{y(b)} \\ V_c &= v_{x(c)}+jv_{y(c)} \end{aligned}\right\} \tag{2}$$

then a current that flows from b to c through the branch bc may be given by :

$$\begin{aligned} I_{bc} &= Y_{bc}(V_b-V_c) \\ &= (g_{bc}+jb_{bc})\{v_{x(b)}+jv_{y(b)}-v_{x(c)}-jv_{y(c)}\} \\ &= [g_{bc}\{v_{x(b)}-v_{x(c)}\}-b_{bc}\{v_{y(b)}-v_{y(c)}\}] \\ &\quad +j[g_{bc}\{v_{y(b)}-v_{y(c)}\}+b_{bc}\{v_{x(b)}-v_{x(c)}\}] \end{aligned}$$

Let the total current flowing into b from all the branches connected to it be

$$-\sum_b I_{bc} = I_{b1} = \{i_{x(b)1}+ji_{y(b)1}\}, \text{ so that,}$$

$$\left.\begin{aligned} -i_{x(b)1} &= \sum_b[g_{bc}\{v_{x(b)}-v_{x(c)}\}-b_{bc}\{v_{y(b)}-v_{y(c)}\}] \text{ and} \\ -i_{y(b)1} &= \sum_b[g_{bc}\{v_{y(b)}-v_{y(c)}\}+b_{bc}\{v_{x(b)}-v_{x(c)}\}] \end{aligned}\right. \quad \ldots \tag{3}$$

Again if $I_{b2} = i_{x(b)2}+ji_{y(b)2}$ stands for the current supplied to b from outside then by Kirchoff's law,

$$\left.\begin{aligned} i_{x(b)} &= i_{x(b)1}+i_{x(b)2} = 0 \\ \text{and} \qquad i_{y(b)} &= i_{y(b)1}+i_{y(b)2} = 0 \end{aligned}\right\} \tag{4}$$

Assuming the vector potential of e to be unity and the points a, b, and c to be at zero potential, the currents flowing from e to b and e to c along the branches eb and ec can be written as ·

$$\begin{aligned} I_{eb} &= Y_{be} = g_{be}+jb_{be} \\ \text{and} \qquad I_{ec} &= Y_{ce} = g_{ce}+jb_{ce} \end{aligned} \tag{5}$$

and no current will flow in any other branch of the circuit. To have the assumed potentials correct, a current, $-I_{e2} = I_{eb}+I_{ec} = i_{x(e)2}+ji_{y(e)2}$ is to be supplied to e from outside Then the currents I_{eb} and I_{ec} will leave the network at b and c respectively But actually no current flows to or from the network at b and c. Hence on the assumed potentials those are to be superposed, which would result if the currents $I_{b(2)}(= I_{eb})$ and $I_{c(2)}(-I_{ec})$ were supplied at b and c and allowed to leave the network at e and a, the latter points being kept at zero potentials. Then it is obtained initially as follows :

$$\begin{aligned} i_{x(b)} &= i_{x(b)2} = g_{be} \,; \quad i_{x(c)} = i_{x(c)2} = g_{ce} \,; \\ i_{y(b)} &= i_{y(b)2} = b_{be} \,; \quad i_{y(c)} = i_{y(c)2} = b_{ce} \,; \\ i_{x(e)} &= i_{x(e)2} = -(g_{be}+g_{ce}) \,; \\ i_{y(e)} &= i_{y(e)2} = -(b_{be}+b_{ce}) \,; \\ \text{with} \qquad i_{x(a)} &= i_{y(a)} = 0. \end{aligned}$$

To liquidate the residuals i.e these initial values $i_{x(b)}$, $i_{y(b)}$, $i_{x(c)}$ and $i_{y(c)}$ by giving suitable vector potentials at b and c only, a standard operation table can be written by the use of the following expression which may be readily obtained from relations (3).

$$\begin{aligned} \frac{\delta i_{x(c)}}{\delta v_{x(b)}} &= g_{bc} = \frac{\delta i_{y(c)}}{\delta v_{y(b)}} \,; \\ -\frac{\delta i_{x(c)}}{\delta v_{y(b)}} &= b_{bc} = \frac{\delta i_{y(c)}}{\delta v_{x(b)}} \,, \\ \text{and} \qquad \frac{\delta i_{x(b)}}{\delta v_{x(b)}} &= \frac{\delta i_{y(b)}}{\delta v_{y(b)}} = -\sum_b (g_{bc}) \,; \\ -\frac{\delta i_{x(b)}}{\delta v_{y(b)}} &= \frac{\delta i_{y(b)}}{\delta v_{x(b)}} = -\sum_b (b_{bc}). \end{aligned} \quad \ldots \ (6)$$

After liquidation of the residuals the vector currents at a and e, and the vector potentials at b and c are obtained, which corresponds to unit potential difference between e and a. In liquidating the residuals first of all a set of operation groups can be found out by considering the operation steps of unit operation table in successive pairs to keep only one residual i e 1st, 2nd etc., unchanged consecutively. From these groups a second set of operation groups are obtained to keep first two residuals i e. 1st and 2nd, unchanged at a time. Lastly, another operation group can be written from the second set of groups which will not have any effect on the values of the first three residuals simultaneously. Using these group operations a definite number of liquidation steps can be obtained not exceeding the number of residuals. Although by this method a quick and systematic liquidation is achieved, it imposes no restriction in finding out suitable operation blocks and other groups if required in any particular problem. Hence its flexibility is not affected.

ILLUSTRATION

An example (Fig. 1) worked out by different method (Mem. Staff Dept Elect Engg. M.I.T, 1953), has been taken as an illustrating one in which $E_{max} = 150 \angle 0$ volts, frequency $= 200$ cycles per second, $R_1 = 12.00$ ohms, $L_1 = 0.300$ henry, $R_2 = 19.20$ ohms, $L_2 = 0.1862$ henry, $R_3 = 250.0$ ohms, $L_3 = 6.50$ henries, $R_L = 200.0$ ohms, $S_L = 6.29 \times 10^5$ darafs

The effective value of the current delivered by the source, effective value of the voltage across cd and its phase relative to that of the source are to be calculated.

Considering the different branches of the network shown in Fig. 2b, their corresponding admittances can be calculated from the supplied data as follows :

$$Y_{ab} = (0.8682-j27.2)10^{-4}\text{mho}$$
$$Y_{be} = (0.036-j1.224)10^{-4} \quad \text{''}$$
$$Y_{bc} = (3.482-j42.44)10^{-4} \quad \text{''}$$
$$Y_{ce} = (6.897+j17.24)10^{-4} \quad \text{''}$$

In order to simplify the calculation and to have higher accuracy of results the above admittances are multiplied by 10^4, keeping in mind that in finding out the currents, the required quantities are to be multiplied by 10^{-4} and also by the effective value of E_{max} of the example.

Hence the currents flowing in the branches ec and eb can be written as :

$$I_{ec} = 6.897+j17.24$$
$$I_{eb} = 0.036-j1.224$$

Then the current of $-I_{e2}$ that is to be flown to e from outside comes out to be.

$$-I_{e2} = 6.933+j16.016$$

So initially it may be obtained as follows :

$$i_{x(b)} = 0.036; \quad i_{x(c)} = 6.897; \quad i_{x(e)} = -6.933;$$
$$i_{y(b)} = -1.224; \quad i_{y(e)} = 17.24; \quad i_{y(c)} = -16.016.$$

and $$i_{x(a)} = i_{y(a)} = 0.$$

Then the unit operation table (Table I) can be written with the help of relations (6) and by multiplying the g and b values by 10^1 for the reason already stated. The required group operations (Dutta, 1966) can be performed to have no change of $i_{x(b)}$, $i_{x(c)}$, $i_{y(b)}$, and $i_{y(c)}$ successively by using unit operation steps 1 and 2, 2 and 3, 3 and 4, and 4, and 1 of Table I, in pairs shown in group operation steps 5, 6, 7 and 8 in Table II From these steps (operation steps 5 to 8 in Table II) two other operation groups (operation steps 9 and 10 in Table II) are found out to keep both $i_{x(b)}$ and $i_{x(c)}$ unchanged simultaneously Finally, another group operation step (operation step 11 in Table II) is developed from those steps (operation steps 9 and 10 in Table II) in which $i_{x(b)}$ $i_{x(c)}$, and $i_{y(b)}$ are seen to remain unchanged at a time. The residuals can now be liquidated very easily and systematically in four steps (liquidation steps 12, 13, 14 and 15 in Table III) using the unit and group operation steps (operation step 1 in Table I and 5, 10 and 11 in Table II) without spending much time for finding out other suitable block or group operation by inspecting the unit operation table. The operation and liquidation steps are shown within the brackets( ) and the actual operation and liquidation are shown within the brackets [ ].

Considering the effective value of E_{max}, the effective value of the voltage across cd(Fig. 1) which is the same as that across ce(Fig. 2b) is given by,

$$V_{cd} = (0.0539 - j2.0961) \times 10.6082 \text{ volts}$$
$$= 22.2433 \angle -88^\circ 32' \text{ volts}\{22.1 \angle -90^\circ \text{ volts}\}$$

and the required current is given by,

$$I_a = (34.9644 - j14.1152) \times 10.6082 \times 10^{-4} \text{ amp}$$
$$= 0.0399 \angle -21^\circ 59' \text{ amp}\{0.0397 \angle -23^\circ 30' \text{ amp}\}$$

From the above it is seen that the values of the voltage and the current calculated by other method (Mem. Staff. Dept. Elect Engg.,M I. T., 1953) shown within brackets { } are in good agreement with those obtained by relaxation method.

DISCUSSION

This paper shows clearly how with the help of the method indicated in the paper, values of a number of desired quantities i.e. potentials at the nodal points b and c, currents at a and e, and the respective phase angles can be obtained simultaneously. This method becomes useful particularly when the number of nodal points increases in complex networks, where the conventional methods are seen to be laborious.

Also in the method used here to liquidate the residuals, a set of operation groups (operation steps 5 to 8 in Table II) as obtained may be found to be useful in keeping any particular residual unchanged in some operation if required.

Further it is seen that V_{cd} is greater than the voltage impressed by the source on the circuit. This is due to the fact that in A C. circuits the impedance of a part may be greater than that of the entire circuit as the imaginary components may be either positive or negative.

ACKNOWLEDGMENT

The author is highly indebted to Prof A. K Sengupta, D Sc, A.M I.E.E. (London), Head of the Department of Applied Physics, Calcutta University, for his help and guidance throughout the progress of this work

REFERENCES

Allen, D N. de G., 1954, *Relaxation Methods* (McGraw-Hill Book Co Inc., New York), 17.

Bandyopadhyay, G and Narsimhan, R K., 1956, *Quart. J. Mech and Applied Math.*, **9**, 122.

Basu, R. N., 1958, *Jour. Assn Applied Physicists*, **30**.

Black, A N and Southwell, R. V., 1938, *Proc Roy. Soc.* (*A*) **164**, 447

Dutta, S. N , 1966, *Indian J. Phys.*, **40**, 1, 1.

Members of the Staff of the Department of Electrical Engineering, Massachusetts Institute of Technology, 1953, *Electric Circuits.* The Technology Press, M I T., 292.

Say, M. G., 1962, *The Performance and Design of A. C. Machines.* Sir Issac Pitman & Sons, Ltd , London, 339.

Southwell, R. V., 1951, *Relaxation Methods in Engineering Science*, Oxford University Press, London, 114